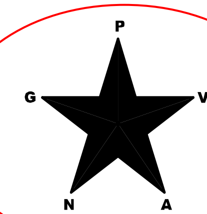
Sample of a teacher page which is included with each lesson in the Laying the Foundation guides

Taken from the Laying the Foundation guide for Middle Grades



Solids of Revolution

The LTF star illustrates the multiple representations utilized in the lesson. A point of the star is shaded if the lesson uses that learning modality. In this lesson,

P (physical): The teacher notes contain instructions for creating hands-on models to represent the questions in the lesson.

V (verbal): Students explain how changing the axis of revolution changes the dimensions and volume of the solid.

A (analytical): Students select appropriate formulas by analyzing the graph.

N (numerical): Students calculate the volumes of the regions.

G (graphical): Students graph and connect the coordinate points.

Updated: 07/01/09

Objective:
Students will calculate volumes of cylinders and cones and combination figures formed by revolving various regions around different axes.

Connections to Previous Learning:
Students should be able to plot points in a coordinate plane, calculate areas, and calculate volumes.

Connections to AP*:
AP Calculus Topic: Areas and Volumes

Materials:
Student activity pages, graph paper, scientific or graphing calculators

Teacher Notes:
Almost all students have difficulty in visualizing 3-dimensional solids. Teachers may want to model the revolutions around different axes by taping a cut-out of the planar region to a pencil, straw or skewer, then rolling the pencil between your hands to simulate the “sweeping out” of the solid. Make sure that students are correctly identifying the radius and the height for each generated solid. Ask students to explain how changing the axis of revolution changes the dimensions of the solid that is formed.

When drawing a picture of a revolution, use the following procedure:

- Draw the boundaries.
- Shade the region to be revolved.
- Draw the reflection (mirror image) of the region about the axis of revolution.
- Connect significant points and their reflections with ellipses.

All LTF lessons are connected to at least one AP topic.

All lessons include the objective and the connections to previous learning. Specific objectives for each state are listed on the website versions.

Selected sample questions from:

Solids of Revolution

General instructions: When calculating volumes of cylinders and cones, give your answer both in terms of π and also as a decimal accurate to three decimal places. Use the π key on your calculator and then round your answer as the last step.

3. a) Draw line segments joining the points (0, 0), (0, 1), and (5, 0).

   b) Calculate the area of the region formed.

   c) Draw and describe the solid formed by revolving the region about the *x*-axis.

   d) Calculate the volume of the solid formed.

   e) Draw and describe the solid formed by revolving the region about the *y*-axis.

   f) Calculate the volume of the resulting solid.

   g) Compare the volume in parts (d) and (f). Explain why these volumes are different.

4. a) Draw line segments joining the points (0, 0), (0, 3), (5, 3), and (5, 0).

   b) Calculate the area of the region formed.

   c) Draw and describe the solid formed by revolving the region about the *x*-axis.

   d) Calculate the volume of the solid formed.

   e) Draw and describe the solid formed by revolving the region about the vertical line $x = 5$.

   f) Calculate the volume of the resulting solid.

Students provide explanations.

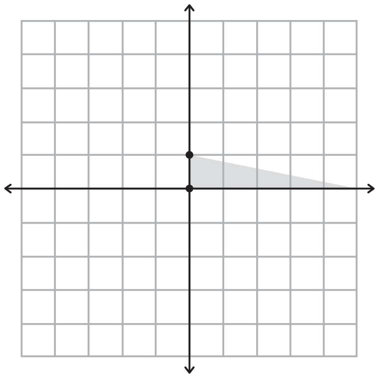
The concept of revolving a plane figure around an axis is not included in most standard textbooks; however, it is a topic that is easily accessible at the middle grades level. Students may need assistance in visualizing the resulting solid, but they already possess the skills necessary to compute the volume of the solid. This extension will greatly improve their level of success in visualizing rotations of curves in calculus.

Selected sample answers from:

Solids of Revolution

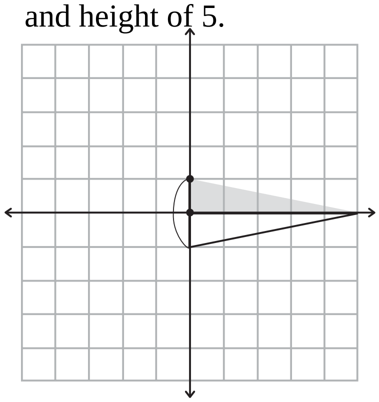
Answers:

3. a)



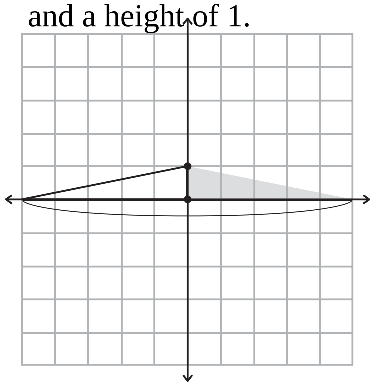
b) $A = \frac{5}{2}\text{ u}^2 = 2.5\text{ u}^2$

c) A cone with a radius of 1 and height of 5.



d) $V = \frac{5}{3}\pi\text{ u}^3 \approx 5.236\text{ u}^3$

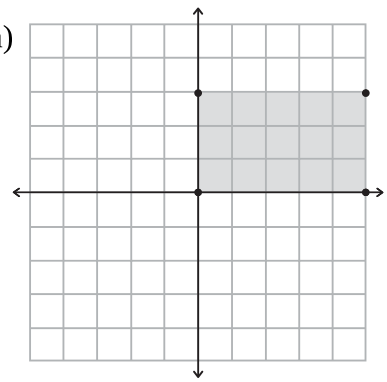
e) A cone with a radius of 5 and a height of 1.



f) $V = \frac{25}{3}\pi\text{ u}^3 \approx 26.180\text{ u}^3$

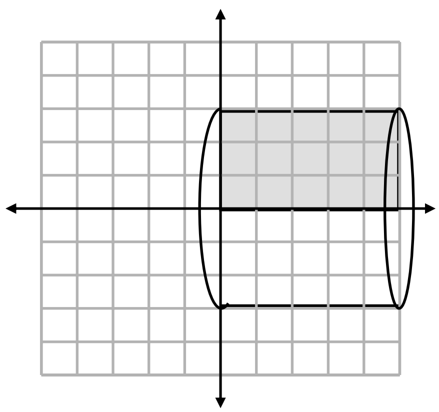
g) The difference in the volumes is $\frac{20}{3}\pi \approx 20.944\text{u}^3$; the radii of the cones are not the same when revolved.

4. a)



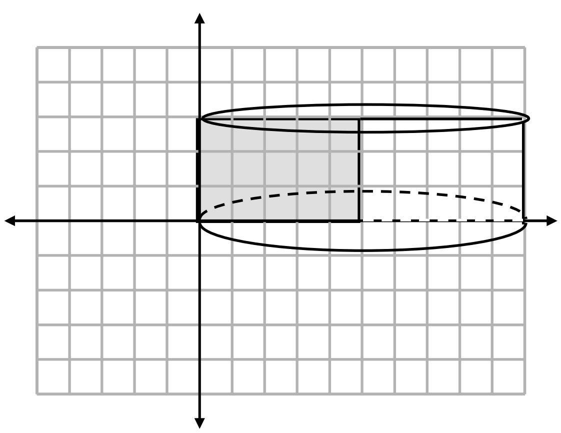
b) $A = 15\,\text{u}^2$

c) A cylinder with a radius of 3 and height of 5.



d) $V = 45\pi\ \text{u}^3 \approx 141.372\ \text{u}^3$

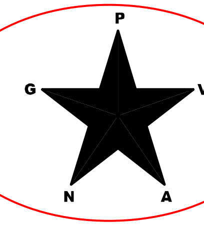
e) A cylinder with a radius of 5 and a height of 3.



f) $V = 75\pi\ \text{u}^3 \approx 235.619\ \text{u}^3$

Taken from the Laying the Foundation guide for Geometry

Sample of a teacher page which is included with each activity in the Laying the Foundation guides



Volumes of Revolution

The LTF star illustrates the multiple representations utilized in the activity. A point of the star is shaded if the activity uses that learning modality. In this activity,

P (physical): The teacher notes contain instructions for creating hands-on models to represent the questions in the lesson.

V (verbal): Students explain how changing the axis of revolution changes the dimensions and volume of the solid.

A (analytical): Students create graphs based on an equation and select appropriate formulas by analyzing the graph.

N (numerical): Students calculate the volumes of the regions.

G (graphical): Students graph linear equations.

Updated: 04/02/09

Objective:
Students will visualize the volume of a geometric solid generated by revolving a bounded region in the plane around the *x*-axis or *y*-axis.

Connections to Previous Learning:
Students should have experience with coordinate graphing, linear functions, graphs of semi-circle and parabolas. In addition, they should be familiar with basic geometric formulas for perimeter, area, volume, and surface area.

Connections to AP*:
AP Calculus Topic: Areas and Volumes

Materials:
Student activity pages

Teacher Notes:
The concept of revolving a region about an axis is fundamental to integral calculus. This lesson includes calculating perimeter and area of a planar region and then the volume generated by rotating the region around the *x*-axis or the *y*-axis.

A suggestion is to glue or tape a triangle on a stick or dowel and rotate the triangle horizontally and vertically so the students can "see" the cone that will be generated.

An extension of this lesson is to bring 3-dimensional objects to class and ask the students to draw cross-sections. Being able to visualize solids is a valuable tool for calculus.

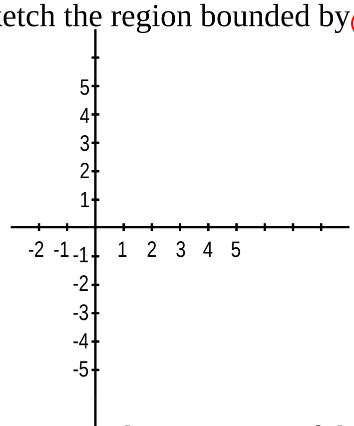
When drawing a picture of a revolution, use the following procedure:

- Draw the boundaries.
- Shade the region to be revolved.
- Draw the reflection (mirror image) of the region about the axis of rotation.
- Connect significant points and their reflections with ellipses.

Selected sample questions from:

Volumes of Revolution

2. a) Sketch the region bounded by the lines $y = \frac{3}{4}x - 3$, $y = 0$, $x = 0$.

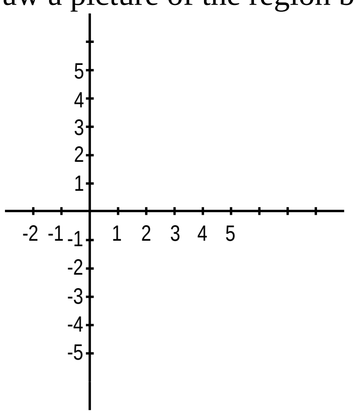


Notice the similarity between questions in this activity and the middle grades activity. The level of difficulty has increased and now includes graphing linear equations.

b) Determine the perimeter of the region.

c) Determine the area of the region.

d) Draw a picture of the region being revolved about the *x*-axis.



As in most LTF lessons, students explain and extend their answers.

e) What geometric figure is formed by revolving the region about the *x*-axis?

f) Determine the volume of the geometric solid.

g) Determine the surface area of the geometric solid.

h) If the region were revolved about the *y*-axis, would the volume be greater than, less than, or equal to the volume formed by revolving about the *x*-axis? Justify your answer. Compare the surface areas.

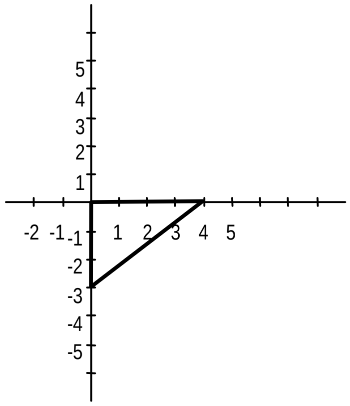
i) Name another region that could be revolved about the *x*-axis to create exactly the same geometric solid.

Selected sample answers from:

Volume of Revolution

Answers:

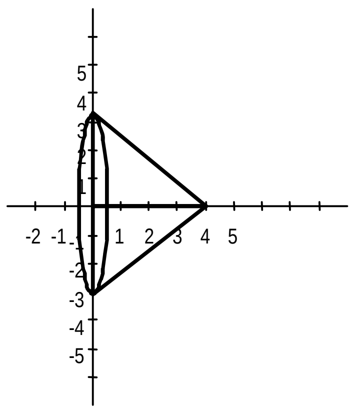
2. a)



b) 12 units

c) 6 sq. units

d)



e) cone

f) 12π cu. units

g) 24π sq. units (remember the base)

h) The volume is 16π cu. units, so the *y*-axis rotation has greater volume. The surface area, 36π sq. units, is greater in the *y*-axis rotation as well.

i) Region bounded by $y = -\frac{3}{4}x + 3$, $y = 0$ and $x = 0$.

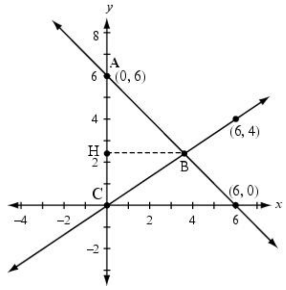
Free Response | Part A

Algebra 1

ΔABC is drawn in quadrant I as shown below where point C is located at the origin.

This question demonstrates how the topic is tested at the Algebra 1 level.

Indicates the question is from the Algebra 1 End of Course Test and was part of the first free response section which allowed the use of calculators



(a) What are the equations of each of the three lines $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, and $\overleftrightarrow{AC}$? Identify each answer.

(b) What are the coordinates of the point of intersection of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$?

(c) What is the length of the altitude, $\overline{BH}$, of ΔABC?

(d) What is the area in square units of ΔABC? Show the work that leads to your answer and state the answer correct to one decimal place.

(e) ΔCHB is revolved about the *y-axis* to form a cone. What is the volume in cubic units of this cone? Show the substitution of the appropriate values into the given formula, and round the final answer to the nearest tenth. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

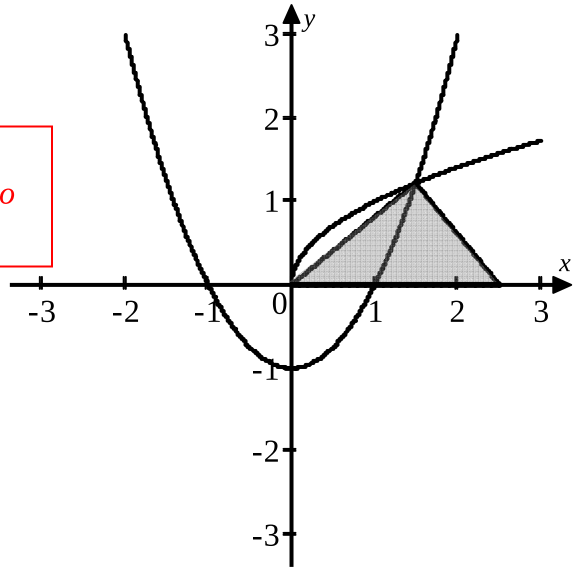
Solution	Scoring
(a) $\overleftrightarrow{AB}$: $y = -x + 6$ $\overleftrightarrow{BC}$: $y = \frac{2}{3}x$ $\overleftrightarrow{AC}$: $x = 0$	(a) 3 pt: 1 pt: correct equation for $\overleftrightarrow{AB}$ 1 pt: correct equation for $\overleftrightarrow{BC}$ 1 pt: correct equation for $\overleftrightarrow{AC}$
(b) $\left(\frac{18}{5}, \frac{12}{5}\right)$ or (3.6, 2.4)	(b) 1 pt: 1 pt: intersection point based on student's answer in part (a)
(c) $BH = 3.6$	(c) 1 pt: 1 pt: student's *x*-value from part (b)
(d) $A = \frac{1}{2}(6)(3.6) = 10.8$ square units	(d) 2 pt: 1 pt: correct base (6) 1 pt: area correct to one decimal based on student's *x*-value from part (b) **or** (c)
(e) $V = \frac{\pi}{3}(3.6)^2(2.4) =$ $32.57203263 \approx 32.6$ cubic units	(e) 2 pt: 1 pt: student's *y* value from part (b) for *h* 1 pt: volume rounded to the tenth's place based student's *x*-value from (b) or altitude from (c) **and** student's *y*-value from (b)

Notice that the point is earned based on the student's answer from part (a). "Reading with" the student is incorporated in parts b, c, d, and e.

Indicates the question was from the Pre-Calculus End of Course Test and was part of the second multiple choice section which allowed the use of calculators

A triangle in the first quadrant has one vertex at the origin, one vertex at (2.5, 0), and the third vertex at the point of intersection of the graphs of $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$. What is the volume of the solid formed when the triangle is revolved about the x-axis?

This question shows the progression of this topic to the pre-calculus level.



(A) 1.526 cubic units
(B) 3.901 cubic units
(C) 5.814 cubic units
(D) 6.227 cubic units
(E) 9.340 cubic units

Rationales provide the explanation for the correct answer choice as well as errors in student thinking that lead to incorrect answer choices.

Rationales:

(A) Area of the triangle is 1.526 square units.
(B) Correct answer.

$$\frac{\pi}{3}(\text{y-value of the intersection point})^2(\text{x-value of the intersection point}) + \frac{\pi}{3}(\text{x-value of the intersection point})^2(2.5 - \text{x-value of the intersection point})$$

$$\frac{\pi}{3}(1.2207441)^2(1.4902161) + \frac{\pi}{3}(1.2207441)^2(2.5 - 1.4902161) = 3.901376679$$

(C) Switched the radius and the height
(D) Used the distance from 0 to 2.5 as the height of the second cone
(E) Used $\frac{\pi}{2}$ in place of $\frac{\pi}{3}$ in volume of the cones